EQUITY DERIVATIVES MINI-MASTER

CONTENTS OF THE PAPER:

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DEFINITION OF VOLATILITY:

Volatility is a measure of the uncertainty of the future price of the underlying asset at the time when the option expires.
There are three types of volatility: historical, implied and realized. Historical volatility measures what standard deviation has been experienced over a given historical period; implied is the value assigned to the volatility in pricing models that equate the modeled price of an option to the market price of that option; realized volatility is the movement in the underlying asset between the date when the option is traded and its expiry.
The assumption of the Black-Scholes model is that the volatility of the underlying asset is constant; therefore the dispersion of asset prices is influenced only by the remaining time to expiry of the option.
In reality volatility changes over time and for different levels of in-the-moneyness.

PROBLEMS WITH THE ESTIMATION OF HISTORICAL VOLATILITY

There are two alternative estimation processes for obtaining historical volatility: the most widely utilized method lays on the assumption that volatility is a fixed parameter and that the historical standard deviation of returns can describe the future probability distribution of the underlying asset. The second approach considers volatility to be a time-varying process and uses advanced econometric techniques (i.e. GARCH) to model the volatility process to be included in the option pricing model.
The calculation of historical volatility raises some important questions:
- how to handle dividends in the volatility calculation? The stock price falls on the ex-dividend date and the amount of the fall attributable to the dividend payment is a function of the net of tax payment received by the holder. It is not possible to determine how much of the price movement is due to the dividend payment and how much to the market because of different marginal tax rates for different investors. One theoretically consistent solution is to ignore the data around the ex-dividend date given that dividends are paid infrequently.
- what is the optimal frequency of the data sampling? We take for granted that the daily observations should be considered, but why? From the perspective of the option writer, a short call option position will be hedged by going long a fraction of a unit of the underlying asset. This fraction is determined by the delta of the option, which will change over time as the price of the underlying stock varies and will determine the need to rebalance the hedge portfolio periodically. As most of option writers review the risk of their portfolios daily, daily data is most commonly used in estimating volatility.
- what type of price should we consider? The last price seems the most satisfactory. Some researchers have suggested that in addition to closing prices also opening prices, daily highs and lows should be considered; the rationale behind it resides on the fact that an intra-daily observation on the stock price movement allows to gather more information on the real risk linked to a security. A stock price behavior, for instance, could look quite stable (thus with a low historical volatility) basing the analysis on closing prices but could be characterized by significant intra-day movements.

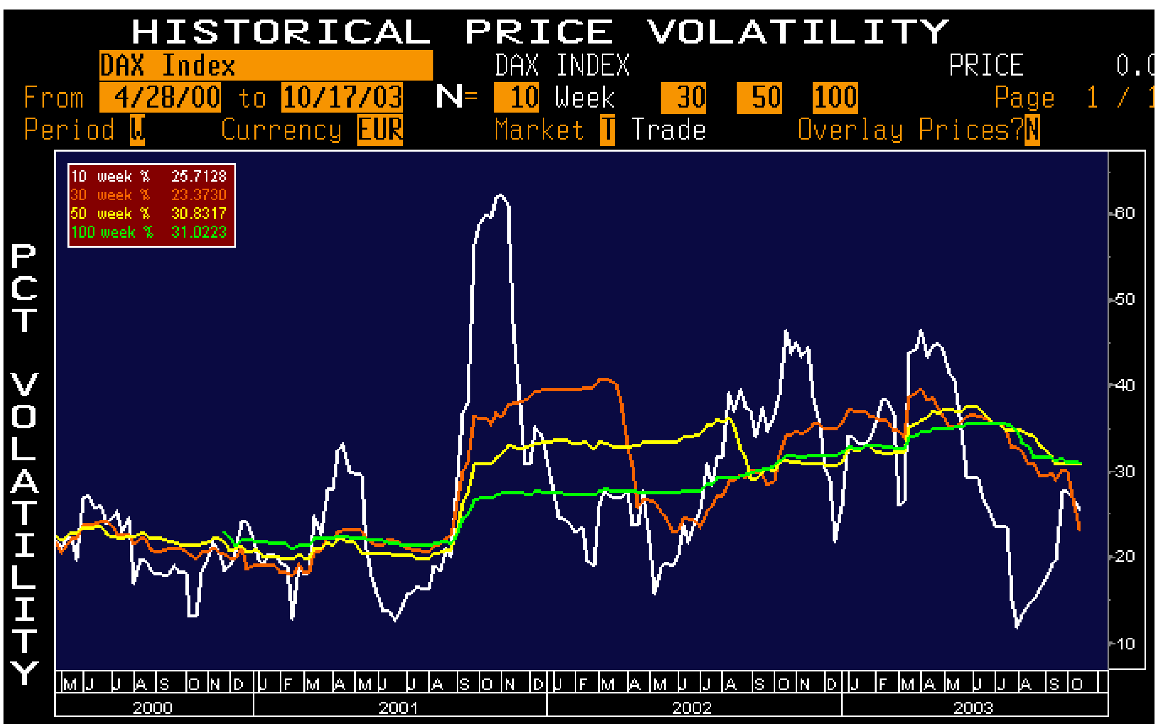
The volatility based on closing prices is to be preferred, having some indubitable advantages such as smaller measurement errors and higher consistency with the assumption of log-normality.

IMPLIED VOLATILITY versus HISTORICAL

The rationale behind the calculation of historical volatility is to use it as either a single forecast of the future volatility or as a valid proxy for it. An alternative is to gather the consensus forecast of the various participants to the option market and to calculate the volatility implied in traded prices of options.
There are some problems also using implied volatility. Firstly, the use of market consensus assumes that the market is strong form efficient. Secondly, it assumes that the option pricing model is a ‘true’ model. In particular, the Black – Scholes approach and the binomial option pricing model make two important assumptions regarding volatility:

- it is know
- it is constant

Unfortunately, empirical observation of both historical and implied volatility show a number of characteristics that are contrary to the assumptions made in the model. The volatility is not known, it must be estimated and the option pricing theory tell us nothing about how to estimate it.
In addition, volatility is non constant over time:



HVG function

The diagram above shows 10, 30. 50 and 100-day historical volatility based on closing prices for the Dax Index over the last three years. It can be easily noticed that volatility is not constant over time and also that large deviations from some average magnitude of

volatility are followed by other large deviations, so there are clusters of high and low volatility. In addition, after such clusters the magnitude reverts to some average longer term feature; this characteristic is called mean-reversion.
Next we notice that a higher level of volatility is associated with falls rather than rises in the level of the underlying variable (stock and stock indices); the explanation of that could reside in :

- the effects of gearing on corporate financial health. When equity markets fall the debt to equity ratio rises, increasing the market's perception of the financial gearing of a company. If a persistent fall is associated with a recession, the operational leverage of a company may deteriorate.
- Empirical studies have witnessed the existence in equity markets of a phenomenon called "herd behavior"; "herding" implies that investors are not fully aware of the dynamics behind the fair pricing of stocks, thus they are driven by the prevailing sentiment in the market. So doing they tend to augment their exposure on equity gradually and progressively following the dominating optimism of other market players (this accumulation process causes low volatility in bullish times). They instead tend to panic and sell off their position in the dramatic attempts to diminish their exposure on equities when there is a hint of recession (this quick sell off causes peaks of volatility)

Mathematically speaking, stock prices and their volatilities are negatively correlated but such correlation is not perfectly inverse (it is not -1 but it usually ranges between -0.3 and -0.6 depending on market dynamics).

Going back to implied volatility, there are some empirical characteristics that is worthy to analyze: skews and smiles, term structure and mean reversion.
The tendency for implied volatility at a given point in time to differ across exercise prices for options with the same expiry is well documented for options on equities, equity indices, interest rates, government bond futures and currencies.
This phenomenon is contrary to the theory of option pricing, because volatility should be a function of the underlying asset, not of the exercise price. Thus, volatility should theoretically be the same for all exercise prices.
The existence of volatility smiles across asset classes indicates that Black-Scholes type models are incorrect, or is incorrect the process generating the stock price (that is commonly assumed to be a Wiener process) and therefore the assumption of log-normally distributed price changes (normally distributed continuously compounded returns).

SKEW function

However one feature of the volatility smiles is that they are not constant (so it is more appropriate to talk about volatility surfaces) over time; in most financial options the smile is greater and more pronounced for short-term options and less visible for longer term options.

The chart above shows 4 volatility smiles for options on 4 different expiry dates on CSCO US Equity. When the options have considerable time to expiry - 6 months, the line highlighted in green- the implied volatility is similar across exercise prices. As expiry approaches (white lines, options with 1 month of residual life) the smile becomes more pronounced.

It can be that the market's assumption on the probability distribution of returns changes as options approach expiry (the market assume more "jumps" in diffusion process as for shorter term options that is assumed by normal distribution).

There are also other reasons for the high implied volatilities of out-of-the money options:

- the high impact of bid-offer spreads on very low-priced options, this impact will be even more evident as the options lose time value (high theta) as expiry approaches
- some empirical studies suggest that roughly only half of the observed smiles in the European stock index option market is due to a smile in the "true" implied volatilities. The remaining part is caused by hedging pressure, deriving in particular from hedgers buying out-of-the money put options to protect their portfolios long of equity against a possible market crash. This purchases raise option prices and therefore implied volatilities of low strike options.

From the screen above (delta points for calls and puts with one month to go on the same stock of the previous example) we can observe another fairly common characteristic of the smile, the "volatility smirk" meaning the smiles are not necessarily symmetric but calls and puts with an equal degree of in-the moneyness or out-of-the moneyness trade at different implied volatilities. This will cause the smile to be either positively or negatively skewed.

The fact that volatility is not constant has important implications for option pricing purposes. The relationship between the implied volatility and the time to expiry (term structure of volatility) may incorporate information about how the market expects volatility to change.   [Please see function TRMS]
Thus "forward volatility" may give information about expectations of future volatility similarly to the way forward interest rates give information about expectation of future interest rates.
The forward volatility can be implied from current implied vols; in order to calculate forward vols we must first work on variances (which are additive whereas volatilities are not), the forward variance is: Vf={[(Tn-t)*Vn]-[(Tm-t)*Vm]}/[Tn-Tm]
Where: t = present time, Tm = expiry date of the shorter term option , Tn = expiry date of the longer term option.

		days to expiry	22	41	59	121	252
csco us equity		atm imp vols	35.275	32.948	34.122	33.993	34.286
		forward vols	#	30.029	36.656	33.870	34.554

AN EXAMPLE OF CALENDAR SPREAD

Directional spread strategies are set up to benefit from directional movement in the underlying asset. When executing these strategies is assumed that the movement will be limited so that the cost of buying an option can be offset by a simultaneous option writing; some examples of directional spreads include:

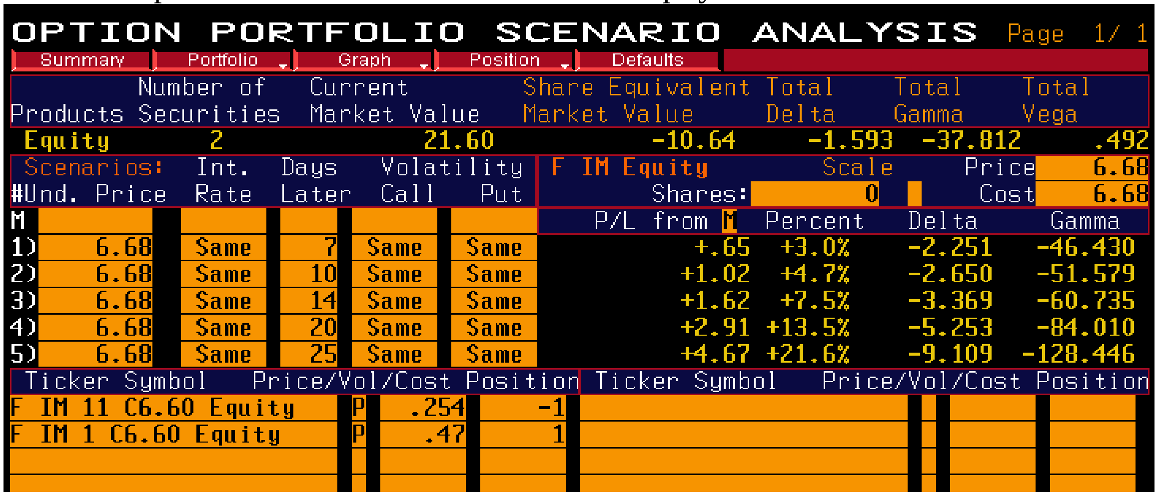
- vertical spreads,
- calendar spreads,
- diagonal spreads.

Calendar spreads entail buying a long-term option and selling a short term option with the same exercise price. This strategy is appropriate if we expect the price of the underlying to remain stable until the short term option expires and then to move upwards.

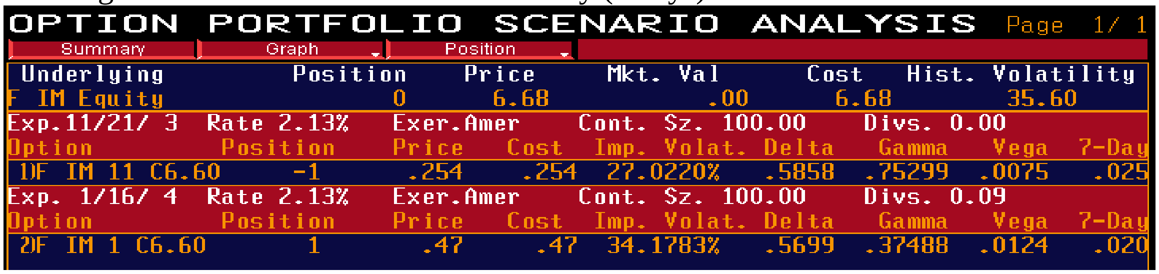
As the theta[1] of the short term written option is greater than that of the long term option purchased, the position should show a profit if the price of the underlying asset does not change, and the implied volatility of the long term option does not fall relative to the volatility of the short term option[2].

However, if the price of the underlying stock is not constant the value of the strategy will be influenced by the aggregate deltas, gammas, thetas and vegas.

As for the following example, we go long 1 position in atm calls with 3 month to expiry and 1 short position in atm calls with 1 month to expiry:



this strategy is opened up for a net cost of 21.60, the net delta of the position is -1.593, the net gamma -37.811 and the net time decay (7 days) is +0.005.



[1] Theta measures the amount at which the time value of options decay as time passes, it is negative for both call and put options meaning that the market value of a long position in whatever plain vanilla option will be constantly eroded by the action of time. More formally, we can think about theta as the first derivative of the option premium with respect to time.

[2] That is analogous to say that forward implied vols should preferably remain upward sloped.

the extrinsic value will fall more rapidly in the shorter dated option.

Scenarios: #	Und. Price	Int. Rate	Days Later	Volatility Call	Put	F IM Equity Shares: P/L from M	Percen
M							
1)	6.68	Same	7	Same	Same	+.65	+3.0%
2)	6.68	Same	10	Same	Same	+1.02	+4.7%
3)	6.68	Same	14	Same	Same	+1.62	+7.5%
4)	6.68	Same	20	Same	Same	+2.91	+13.5%
5)	6.68	Same	25	Same	Same	+4.67	+21.6%

and

OPTION PORTFOLIO SCENARIO ANALYSIS Page 1/ 1

Summary | Graph | Position

25 Days From Today   Interest Rates Same
Call Volatility Same   Put Volatility Same

Underlying	Position	Price	Init. Price	Profit/Loss	Accum. Dividend
F IM Equity	0	6.68	6.68	.00	0.00

Exp.11/21/ 3 Rate 2.13% Exer.Amer Cont. Sz. 100 Divs. 0.00

Option	Position	Price	Init Prc	Profit/Loss	Imp. Volat.	Delta	Gamma
1)F IM 11 C6.60	-1	.131	.254	+12.30	27.0218%	.6576	1.7390

Exp. 1/16/ 4 Rate 2.13% Exer.Amer Cont. Sz. 100 Divs. 0.09

Option	Position	Price	Init Prc	Profit/Loss	Imp. Volat.	Delta	Gamma
2)F IM 1 C6.60	1	.394	.47	-7.62	34.1783%	.5665	.45450

we can also see that if the stock price remains unchanged over time the calendar spread will produce significant gains (that progressively rise when approaching the expiry of the shorter option; + 3% if the stock price remains unchanged in a 7-day scenario, +21.6% gain if we hypothise the same scenario in 25 days ahead from now).

The longer term option will be more sensitive in changes in volatility (being its vega higher than the shorter term option):

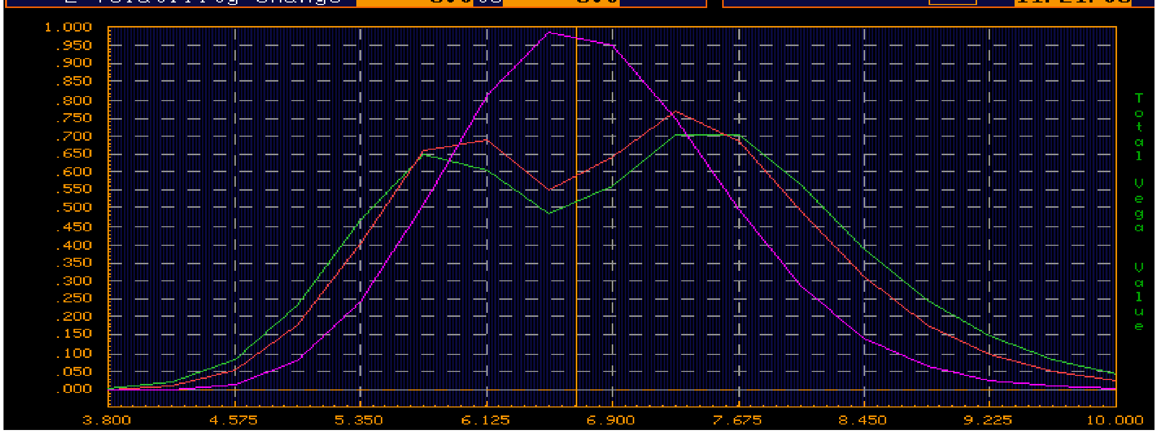
- we can notice that at expiry of the first option the vega of the strategy is bell-shaped (as it should normally be for a long call):

OPTION PORTFOLIO SCENARIO ANALYSIS Page 1/

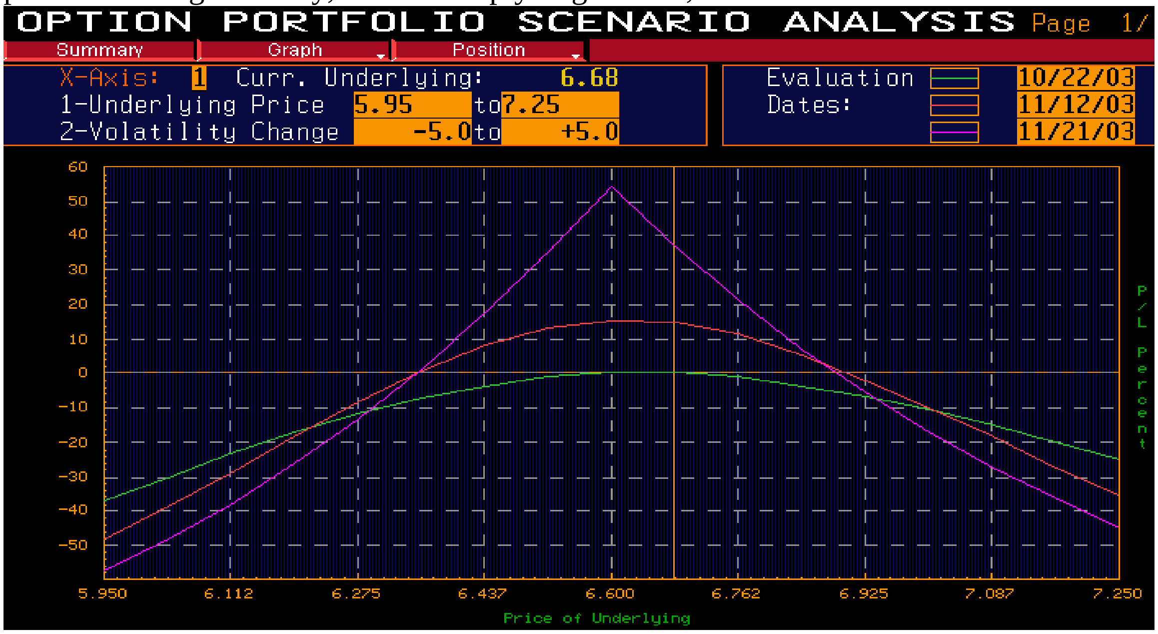
Summary | Graph | Position

X-Axis: 1 Curr. Underlying: 6.68
1-Underlying Price 3.8 to 10
2-Volatility Change -5.0 to +5.0

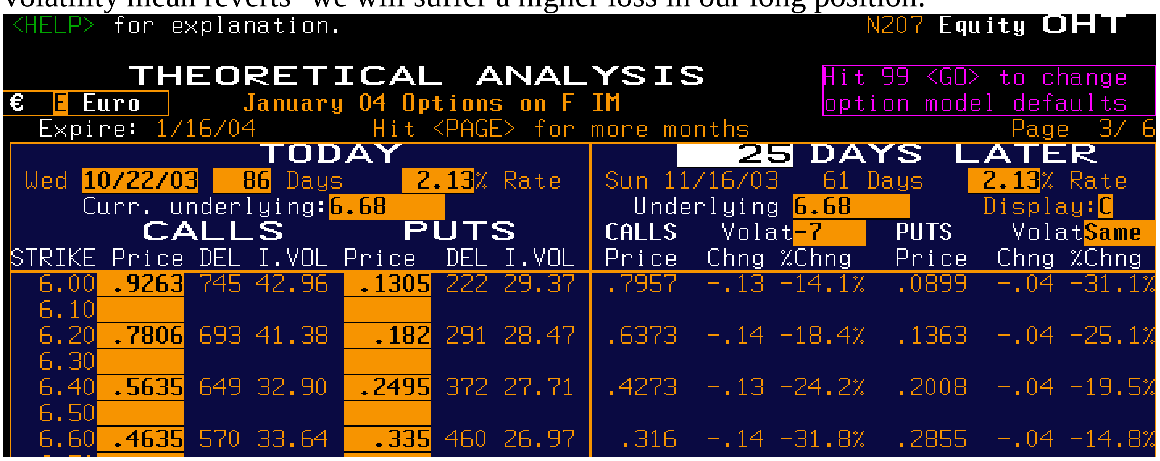
Evaluation Dates: 10/22/03, 11/ 1/03, 11/21/03



In conclusion, the strategy detailed above is particularly leveraged and risky. If the stock price moves significantly, this will imply large losses,



another risk in this particular case derives from the fact that the volatility term structure is upward sloped (so that short term volatility, 27% is lower than long term one, 34%). In fact if, considering again a 25-day scenario with constant underlying price, the long term volatility mean reverts[3] we will suffer a higher loss in our long position:



<HELP> for explanation. N207 Equity OMT

THEORETICAL ANALYSIS

Hit 99 <GO> to change option model defaults

€ E Euro January 04 Options on F IM

Expire: 1/16/04 Hit <PAGE> for more months Page 3/ 6

TODAY: Wed 10/22/03 86 Days 2.13% Rate; Curr. underlying: 6.68

25 DAYS LATER: Sun 11/16/03 61 Days 2.13% Rate; Underlying 6.68; Display: C; CALLS Volat -7; PUTS Volat Same

STRIKE	CALLS Price	DEL	I.VOL	PUTS Price	DEL	I.VOL	CALLS Price	Chng	%Chng	PUTS Price	Chng	%Chng
6.00	.9263	745	42.96	.1305	222	29.37	.7957	-.13	-14.1%	.0899	-.04	-31.1%
6.10												
6.20	.7806	693	41.38	.182	291	28.47	.6373	-.14	-18.4%	.1363	-.04	-25.1%
6.30												
6.40	.5635	649	32.90	.2495	372	27.71	.4273	-.13	-24.2%	.2008	-.04	-19.5%
6.50												
6.60	.4635	570	33.64	.335	460	26.97	.316	-.14	-31.8%	.2855	-.04	-14.8%

the projected price of the long call will fall to 0.316 due to the decline in volatility (otherwise we would expect a value of 0.394) and from an expected profit of 21.6% our strategy will show a 14% loss in 25 days from now.

[3] Falling of 7% to 27%.

AN EXAMPLE OF ARBITRAGE SPREAD

In many markets there are not market-traded futures on individual equities and this can represent a problem for the fund manager whishing to freeze the recent gains experienced on a particular stock.

Of course, for blue chips the problem could be overcome by shorting a suitable number of futures on the index the share is part of. This type of hedging provokes three major problems:

- an initial margin has to be paid and the value of the short position is daily market to market; this is balanced pretty well by the movement in the opposite direction of the underlying stock but whereas the gain on the stock is only potential (cashed when the long position is liquidated) the cost of settling daily margin on the future is real. This causes the hedge to be costly;
- it is not possible to go short a fraction of the future contract. The effect is an annoying over-hedging or under-hedging and different exposures on the cash and future securities;
- the equity edge is a "beta" hedge. This entails that the optimal number of futures to be shortened is calculated by considering how the stock moves compared to the index underlying the future. Beta is calculated on historical time series and there is no certain recipe on how long the data period one must consider and with which frequency. This is the probably the major drawback in using futures on indices.

It is also true that some stocks have individual futures officially traded on them, with 3 or for maturities. The most significant problem this time is represented by the lack of liquidity in the future market:

Related Functions | Contracts | Contract Table

BARCLAYS PLC - Contract Table

Security	Exp	Bid	Ask	Last	Change	High	Low	Volume	OpenInt
1) BARC LN Equity		511.000	511.250	511.000	-16.000	527.250	508.000	31627824	N.A.
2) BARC=X3 LN Equity	Nov03	511.500	513.500	517.00	-11.25	517.00	517.00	2	4
3) BARC=Z3 LN Equity	Dec03	512.750	515.000	529.75	unch	N.A.	N.A.	1	34
4) BARC=F4 LN Equity	Jan04	N.A.	N.A.	531.75	unch	N.A.	N.A.	0	0
5) BARC=H4 LN Equity	Mar04	N.A.	N.A.	520.25	unch	N.A.	N.A.	0	0

as it can be seen above.

The hedging problem can be solved by using equity options.

The underlying stock has a symmetric payoff and the extent of the gain or loss of a pure long position in equity will be determined by multiplying the price of the stock for the number of shares held and again for the delta of the stock.

The gamma of cash equities is zero, this means that their delta does not vary as the stock price changes.

Vanilla options have an asymmetric payoff[4] and must be combined to create a symmetric payoff. The strategy to be created is called short synthetic forward and can be achieved by buying a put and selling a call.

The hedge has the main advantage to be almost costless; instead if we wanted only to insure the long equity position by buying put options there would be the entire put premium to pay.

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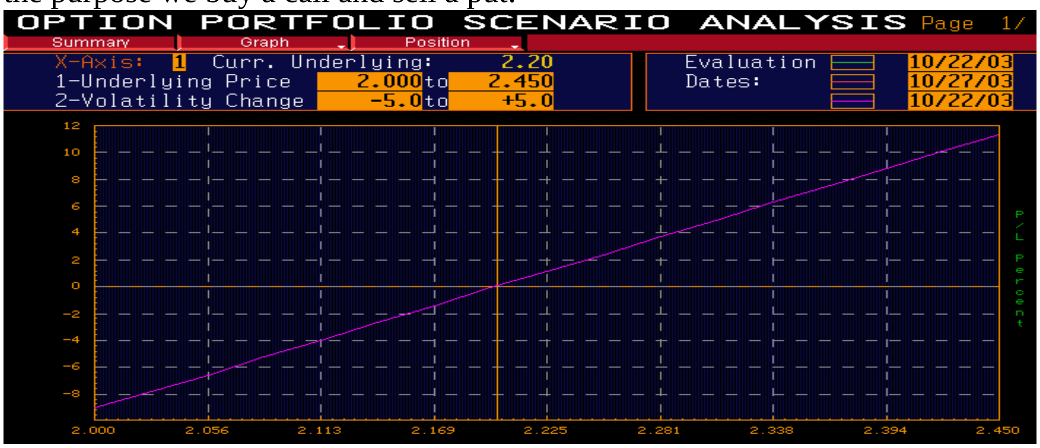
[4] Because their delta is not a constant and their gamma diverges from zero.

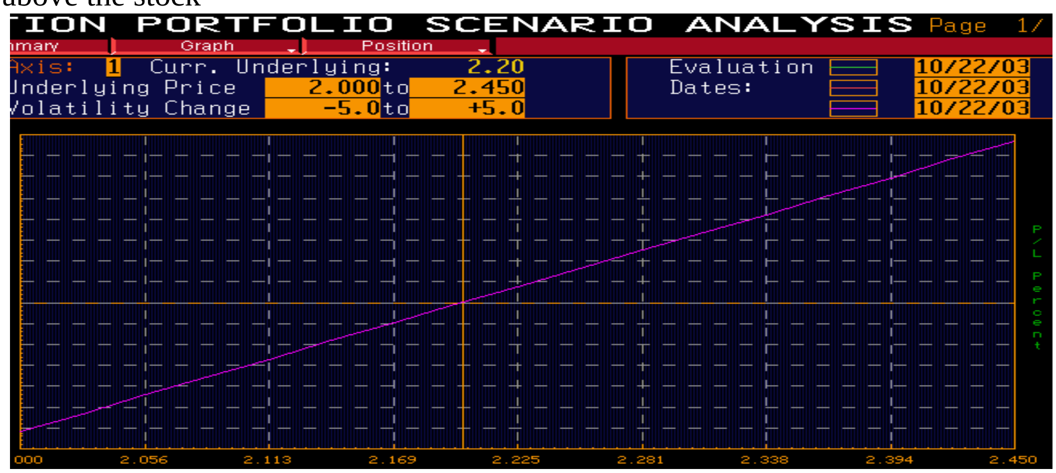
To illustrate how to create a consistent long one-month synthetic forward, we consider at-the money options expiring in one month. The at-the moneyness has two important advantages:

- the difference in option premiums will be least, thus the hedge will be self-financing and the time-decay will cause minimum damage to the synthetic position;
- the opposite position in such similar options will minimize the aggregate gammas and vegas of the portfolio, assuring the goodness of the hedge for large movements of the underlying stock.

We create first a long synt. forward in order to show how well it replicated the stock. For the purpose we buy a call and sell a put:



above the stock

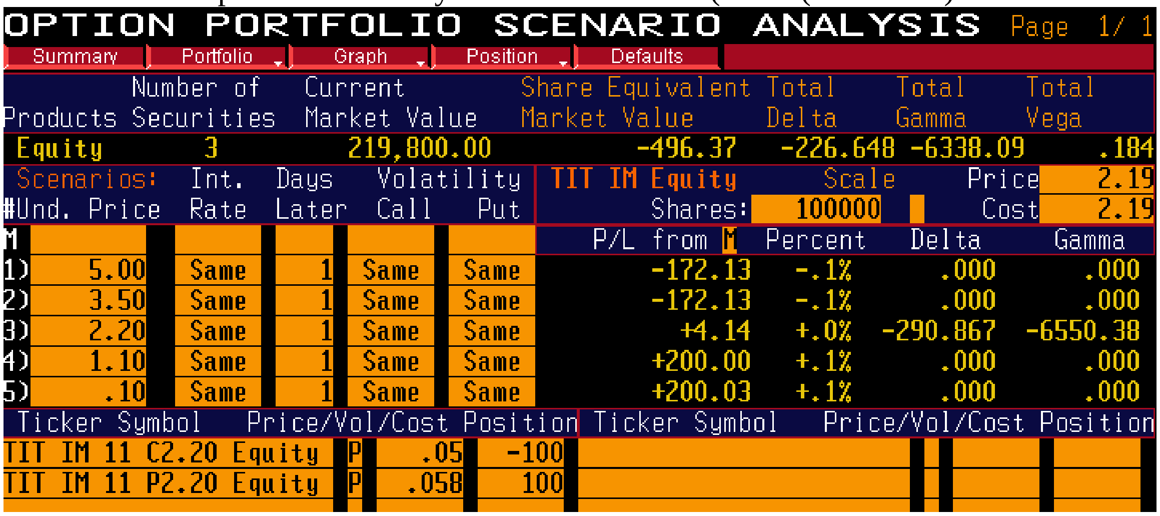


and the synthetic forward.

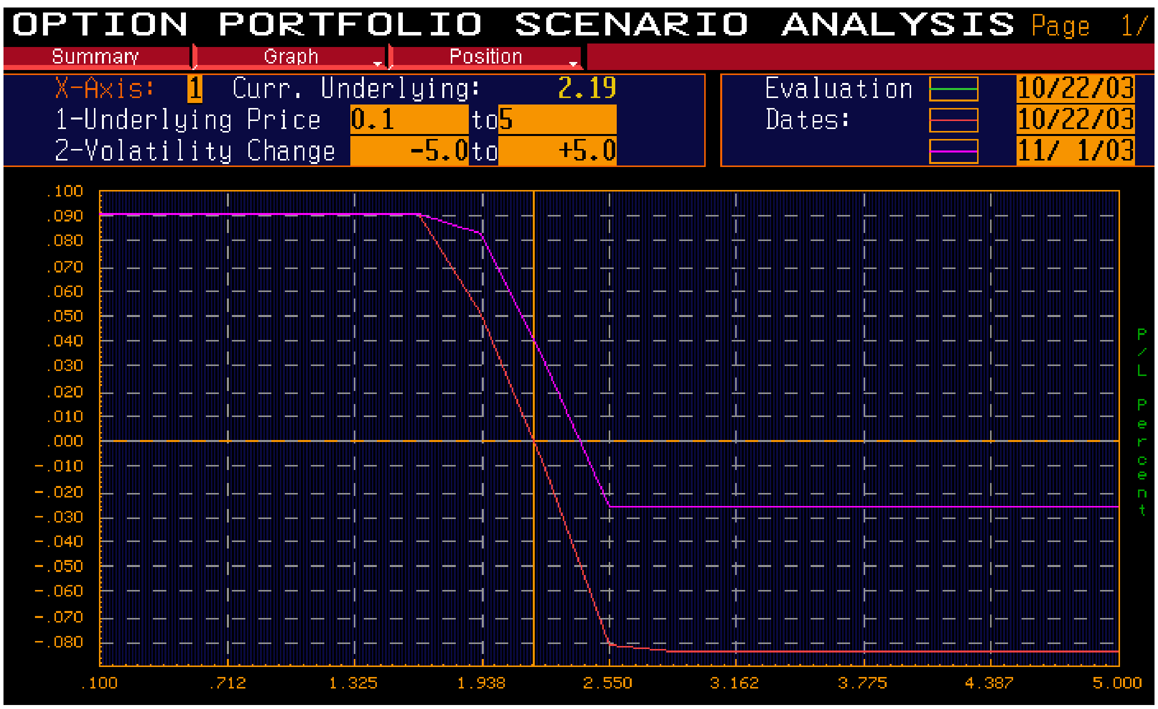
Then we hedge the position:

- we are initially equity long of 100000 TIT IM Equity, whose closing price is 2.19 euro,
- we hedge by buying/selling the same number of puts/calls. The number of calls to sells equals the puts to buy and is given by: - [total delta of the long stock] / (number of shares depending on the exercise on 1 option * (- delta call + delta put)

Being the price of the equity 2.19 we choose 2.20 as strike and observe in OMON the delta last of the options, the delta call is +0.50 and the delta put is also -0.50.
The number of options to both buy and sell is -100000/(1000*(-0.50-0.50) = 100.



and



it can be seen from the 2 screens above that the hedge created has a terrific performance. If we hypothise that tomorrow the price of the equity falls by 95% the hedge will show an error of 0.09% (we would have pocketed 200 euros and insured a long position worth 219000 euros), if the price doubles the error will be 0.08% and we would have lost 185 euros.

Finally, from the detail of the portfolio of options, we can appreciate the almost perfect offsets in gamma, vega and decay:

OPTION PORTFOLIO SCENARIO ANALYSIS Page 1/ 1

Summary | Graph | Position

Underlying	Position	Price	Mkt. Val	Cost	Hist. Volatility
TIT IM Equity	100000	2.19	219000.00	2.19	21.46

Exp.11/21/ 3 Rate 2.13% Exer.Amer Cont. Sz. 1000.00 Divs. 0.00

Option	Position	Price	Cost	Imp. Volat.	Delta	Gamma	Vega	7-Day
1)TIT IM 11 C2.20	-100	.05	.05	21.1409%	.4936	3.0052	.0025	.007
2)TIT IM 11 P2.20	100	.058	.058	21.7487%	-.5086	2.9419	.0025	.006